## Hooke-Jeeves method

1. Apply three steps of the Hooke-Jeeves method with the initial approximation $x = 0$ and $y = 0$ and an initial step size of $h = 1$ to find the minimum of the function

$$f(x, y) = 3x^2 + 5y^2 + 4xy + 17x - 13y + 4$$

Answer: We interpret this as a scalar valued function of a vector variable, but for simplicity, we will work with this function as is for clarity. We observe that:

$$f(-1, 0) = -10, f(0, 0) = 4, f(1, 0) = 24$$

$$f(-1, -1) = 12, f(-1, 0) = -10, f(-1, 1) = -22$$

Now we move in the direction (–1, 1):

$$f(-2, 2) = -40, f(-3, 3) = -50, f(-4, 4) = -52, f(-5, 5) = -46.$$

Thus, we try again at (–4, 4)

$$f(-5, 4) = -58, f(-4, 4) = 52, f(-3, 4) = 40$$

$$f(-5, 3) = -60, f(-5, 4) = -58, f(-5, 5) = -46$$

Thus, we move in the direction of (–1, –1):

$$f(-6, 2) = -44$$

There is no further improvement, so we try again at this new point:

$$f(-6, 3) = -56, f(-5, 3) = -60, f(-4, 3) = -58$$

$$f(-5, 2) = -52, f(-5, 3) = -60, f(-5, 4) = -58$$

There is no change, so we halve $h$ and try again:

$$f(-5.5, 3) = -58.75, f(-5, 3) = -60, f(-4.5, 3) = -59.75$$

$$f(-5, 2.5) = -57.25, f(-5, 3) = -60, f(-5, 3.5) = -60.25$$

Thus, we move in the direction of (0, 0.5):

$$f(-5, 4) = -58$$

There is no further improvement, so we try again at this new point:

$$f(-5.5, 3.5) = -60, f(-5, 3.5) = -60.25, f(-4.5, 3.5) = -59$$

$$f(-5, 3) = -60, f(-5, 3.5) = -60.25, f(-5, 4) = -58$$

Now, at this point, you may be asking: aren't we re-calculating a number of points? Yes, and function evaluations could be reduced by using a look-up table (a map). However, in higher dimensions with a more non-quadratic function, such additional searches may be necessary.

There is no change, so we halve $h$ and try again:

$$f(-5.25, 3.5) = -60.3125,\ f(-5.5, 3.5) = -60.25,\ f(-4.75, 3.5) = -59.8125$$

$$f(-5.25, 3.25) = -60.25,\ f(-5.25, 3.5) = -60.3125,\ f(-5.25, 3.75) = -59.75$$

Thus, we move in the direction of (–0.25, 0), we note that $f(-5.5, 3.5) = -60.25$, so we are done.

As we have finished three iterations, first with $h = 1$, then $h = 0.5$, and then $h = 0.25$, our best estimation of the minimum is at (–5.25, 3.5) where the function has a value of –60.3125. The actual minimum is closer to (–5.04545, 3.31818) where the function has a value closer to –60.454545.

2. Apply three steps of the Hooke-Jeeves method with the initial approximation $x = 1$, $y = 1$ and $z = 1$ and an initial step size of $h = 1$ to find the minimum of the function

$$f(x, y, z) = 4\cos(0.3xy) + 3\cos(0.2yz) + 3\cos(0.1xz)$$

Answer: Without showing the calculations, our first step is to move in the direction (1,1,1), so we see:

$$f(2, 2, 2) = 6.302734128$$
$$f(3, 3, 3) = -2.433064947$$
$$f(4, 4, 4) = -2.732486960$$

At this point, $f(5, 5, 5)$ is greater, so we try again, and we see we should move in the direction (–1, 0 0), and so

$$f(3, 4, 4) = -5.494844728$$

At this point, $f(2, 4, 4)$ is greater, so we try again, but there is no direction of better increase, so we halve the value of $h$. We then try again to see that we should move in the direction (0, –0.5, 0.5)

$$f(3, 3.5, 4.5) = -6.342732549$$
$$f(3, 3\ , 5\ ) = -6.374054453$$

You may wonder why, at the previous step, we did not move in the direction (0, –1, 1); however, as it turns out, $f(3, 4, 4) < f(3, 3, 4)$, so there was no opportunity to try $f(3, 3, 5)$. Anyway, we try again, and we see that we must now move in the direction (0.5, 0, 0.5):

$$f(3.5, 3, 5.5) = -8.002828690$$
$$f(4,\ 3, 6\ ) = -8.489490060$$

Trying again, we see we must move in the direction:

(0, –0.5, 0.5) to get $f(4, 2.5, 6.5) = -9.513025274$
(0.5, 0, 0) to get $f(4.5, 2.5, 6.5) = -9.803830912$

Now, halving $h$ again, we continue moving in the direction:

(0, –0.25, 0.25) to get $f(4.5, 2.25, 6.75) = -9.945872497$
(0.25, 0, 0) to get $f(4.75, 2.25, 6.75) = -9.969134843$

As you may suspect, the actual minimum has a value of –10.